nag_fresnel_s (s20acc)

1. Purpose

nag_fresnel_s (s20acc) returns a value for the Fresnel Integral $S(x)$.

2. Specification

```
#include <nag.h>
#include <nags.h>

double nag_fresnel_s(double x)
```

3. Description

This function evaluates an approximation to the Fresnel Integral

$$S(x) = \int_0^x \sin\left(\frac{\pi}{2}t^2\right)\, dt.$$

The function is based on Chebyshev expansions.

4. Parameters

x

Input: the argument x of the function.

5. Error Indications and Warnings

None.

6. Further Comments

6.1. Accuracy

Let δ and ϵ be the relative errors in the argument and result respectively.

If δ is somewhat larger than the ***machine precision*** (i.e., if δ is due to data errors etc.), then ϵ and δ are approximately related by $\epsilon \simeq |x \sin(\pi x^2/2)/S(x)|\, \delta$.

However, if δ is of the same order as the ***machine precision***, then rounding errors could make ϵ slightly larger than the above relation predicts.

For small x, $\epsilon \simeq 3\delta$ and hence there is only moderate amplification of relative error. Of course for very small x where the correct result would underflow and exact zero is returned, relative error-control is lost.

For moderately large values of x, $|\epsilon| \simeq |2x \sin(\pi x^2/2)||\delta|$ and the result will be subject to increasingly large amplification of errors. However, the above relation breaks down for large values of x (i.e., when $1/x^2$ is of the order of the ***machine precision***); in this region the relative error in the result is essentially bounded by $2/\pi x$.

Hence the effects of error amplification are limited and at worst the relative error loss should not exceed half the possible number of significant figures.

6.2. References

Abramowitz M and Stegun I A (1968) *Handbook of Mathematical Functions* Dover Publications, New York ch 7 p 300.

7. See Also

nag_fresnel_c (s20adc)

8. Example

The following program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

8.1. Program Text

```
/* nag_fresnel_s(s20acc) Example Program
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 2 revised, 1992.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

main()
{
  double x, y;

  /* Skip heading in data file */
  Vscanf("%*[^\n]");
  Vprintf("s20acc Example Program Results\n");
  Vprintf("     x          y\n");
  while (scanf("%lf", &x) != EOF)
    {
      y = s20acc(x);
      Vprintf("%12.3e%12.3e\n", x, y);
    }
  exit(EXIT_SUCCESS);
}
```

8.2. Program Data

```
s20acc Example Program Data
             0.0
             0.5
             1.0
             2.0
             4.0
             5.0
             6.0
             8.0
             10.0
             -1.0
             1000.0
```

8.3. Program Results

```
s20acc Example Program Results
     x          y
   0.000e+00   0.000e+00
   5.000e-01   6.473e-02
   1.000e+00   4.383e-01
   2.000e+00   3.434e-01
   4.000e+00   4.205e-01
   5.000e+00   4.992e-01
   6.000e+00   4.470e-01
   8.000e+00   4.602e-01
   1.000e+01   4.682e-01
  -1.000e+00  -4.383e-01
   1.000e+03   4.997e-01
```